The goal of this learning seminar is to learn the basics of Arakelov geometry, also known as arithmetic intersection theory. Ideally, we will be able to treat Vojta's proof of the Mordell conjecture [Voj91] by the end of the semester, among other things. The general plan is to begin with the theory of intersections on arithmetic surfaces, then moving to the generalization to higher-dimensional varieties developed by [GS90], and finishing with Vojta's proof. A tentative schedule/outline for the seminar is as follows:

1. Metrized line bundles & algebraic preliminaries. 1-dimensional Arakelov theory. Sample references: [Lan88, Ch I], [Mor14, Ch 1, 3], [Neu99, Ch III]

2. Analytic preliminaries. Green's functions and some Riemann surface theory. Sample references: [Lan88, Ch II], [Mor14, Ch 1, 4], [GH94, Ch 0, 1]

3. Intersection theory on arithmetic surfaces. Arithmetic Chow groups, Hodge index theorem, adjunction formula, Faltings–Riemann–Roch, determinants. Sample references: [Lan88, Ch III-V], [Mor14, Ch 4], [Fal84]

4. Intersection theory on higher-dimensional arithmetic varieties (perhaps sketched for sake of time). Arithmetic Riemann–Roch and Hilbert–Samuel. Sample references: [Mor14, Ch 5], [GS90], [SABK92]

5. Proof of the Mordell conjecture, following [Voj91]. Sample references: [Voj93]. A simplified version of the proof due to Bombieri (but more or less avoiding the Arakelov theory) can be found at [BG06, Ch 11].

Below is a starter list of references for this material that might be useful. It'll be updated over the course of the seminar.

References

[Ara74] Suren Arakelov. Intersection theory of divisors on an arithmetic surface. *Mathematics of the USSR-Izvestiya*, 8(6):1167, dec 1974.

[Ara75] Suren Arakelov. Theory of intersections on the arithmetic surface. In *Proceedings of the International Congress of Mathematicians (Vancouver, BC, 1974)*, volume 1, pages 405–408, 1975.

[BG06] Enrico Bombieri and Walter Gubler. *Heights in Diophantine Geometry*. New Mathematical Monographs. Cambridge University Press, 2006.

[Chi86] Ted Chinburg. *An Introduction to Arakelov Intersection Theory*, pages 289–307. Springer New York, New York, NY, 1986.

[Fal84] Gerd Faltings. Calculus on arithmetic surfaces. *Annals of Mathematics*, 119(2):387–424, 1984.

[Fal91] Gerd Faltings. Diophantine approximation on abelian varieties. *Annals of Mathematics*, 133(3):549–576, 1991.

[FZ92] Gerd Faltings and Shouwu Zhang. *Lectures on the Arithmetic Riemann-Roch Theorem. (AM-127)*. Princeton University Press, 1992.

[GH94] Phillip Griffiths and Joseph Harris. *Principles of algebraic geometry*. Wiley Classics Library. John Wiley & Sons, 1994.

[GS90] Henri Gillet and Christophe Soulé. Arithmetic intersection theory. *Publications Mathématiques de l'IHÉS*, 72:93–174, 1990.

[Lan88] Serge Lang. *Introduction to Arakelov Theory*. Springer New York, NY, 1988.

[Mor14] Atsushi Moriwaki. *Arakelov Geometry*. Translations of Mathematical Monographs, Volume 244. American Mathematical Society, 2014.

[Neu99] Jürgen Neukirch. *Algebraic Number Theory*. Grundlehren der mathematischen Wissenschaften. Springer Berlin, Heidelberg, 1999.

[SABK92] C. Soulé, D. Abramovich, J. F. Burnol, and J. K. Kramer. *Lectures on Arakelov Geometry*. Cambridge Studies in Advanced Mathematics. Cambridge University Press, 1992.

[Sou21] Christophe Soulé. *Chapter I: Arithmetic Intersection*, pages 9–36. Springer International Publishing, Cham, 2021.

[Voj91] Paul Vojta. Siegel's theorem in the compact case. *Annals of Mathematics*, 133(3):509–548, 1991.

[Voj93] Paul Vojta. *Applications of arithmetic algebraic geometry to diophantine approximations*, pages 164–208. Springer Berlin Heidelberg, Berlin, Heidelberg, 1993.

[Zha92] Shouwu Zhang. Positive line bundles on arithmetic surfaces. *Annals of Mathematics*, 136(3):569–587, 1992.